An algorithm to determine primes within IN

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1 abstract

I like to show in this article a new algorithm based on combinatorial mathematics, that offers a method to determine the distribution of primes in N, to compute new primes on a finite base of primes, and to proof primality, both on small bases and just using multiplication. A combinatorial formula with a simple method to generate disjunct subsets of N which are definitely multitudes is given. Ordered by size, these products cover N succeedingly, and leave gaps for other multitudes or for further primes. These gaps are unique for a determinable interval, and narrow for the rest. The gaps that are necessarily prime are situated between the first gap in the ordered structure of the set of results $M$and the gap that fits the double of this position, hence the p_{n+1} gap is the first prime gap in M, and the successors are prime until the gap for the multitude $2 \cdot p_{(n+1)}$ Every further step of the algorithm's computation with a rising number of primes allows a more precise identification of a rising number of prime-gaps in distinction to multiple-gaps.

Additionally, the computational algorithm can be used to prove primality with analytical confirmation, as with the disjunctivity of the subsets the cardinality of results must be equal to the results of the usual combinatorial computation implied. Hence, if a computation of all primes that are smaller than the prime-to-be-proved are used within the computation, the result is absolutely secure. But the formula offers additionally a probabilistic proof on the base of subsets - and this is just the state of the art. As the formula shows already a specific structure and method to identify all primes in N, further simple method to distinguish the gaps of primes and gaps of multitudes should be a question of refinements. Additionally, the algorithmic formula shows an argument for the finiteness of primes. But, though puzzlement about Euclid was perhaps the genuin reason to start this line of thought, this result seems less interesting than the deep insight into N's structure and generativity on primes.

2 introduction

We know, that N consists completely of primes and multitudes of primes.
Hence, N is a sum of two disjunct sets, that can very informally and heuristically be defined as such:
$P :=$ { p: p is a prime }
and $M :=$ { m: m is a multitude of at least two primes}.
This structure is genuinely combinatorial. We can obviously use the means of a combinatorial calculus to determine the relation of primes and the non-prime naturals (I use the expression 'natural' to refer to natural numbers) that can be generated by primes. Properly defined it will help us 1) to compute easily the next larger prime on a given set of known primes, 2) to prove with this algorithm whether a collection of succeeding primes are entirely primes or entail a pseudo-prime or a multitude, 3) to compute the relative frequency of primes and 4) to show with the algorithm that the total number of primes is finite, at least with respect to the laws of analysis, as the number of natural numbers that can succeed within the algorithm as a prime converges to 0.

To get a better surview of the magnitudes of primes in relation to N, we can check how many naturals that are multitudes can be generated by multiplication of a respective number of primes. The main idea is to get a set of multitudes, with products that mirrow all possible combinations of primes as factors. As multiplication is commutative, order is not taken into account, but as multitudes of one and the same prime are trivially different, repetition is allowed. Our combinatorial factor k starts with 1 (we could start with 0 to involve 1 into our set M, but I favor a restritive handling of the concept of multitude and keep the 1 outside). K ends with n, so that powers of primes up to the cardinality n $= ||P||$ are allowed.
The main advantage of the combinatorial method is the disjunctiveness of the sets that emerge by the combination without order and with repetition. As combinatorial computation is mathematically already well known, we can make use of the knowledge we have about this in general for the determination of primes.

The elements of the disjunctive sets are products based on these combinations, the combinations base on primes and are limited by the chosen cardinality of primes. The primes shouldn't have any gaps, but can be generated such that they fulfill this demand.

The obvious advantage to the usual handling of the relative frequency of primes is hence the non-arbitrariness of multitudes. This fits the fact concerning N, that $P \cup M = N \setminus \{0, 1\}$, and that we have a proportion of primes to multitudes in N that is not arbitrary, but due to the combinatorial possibilities of numbers and their products.
The usual estimation of the relative frequency of primes, is not just empirically insufficiently confirmed with a horrible lot of just-probable primes or pseudo-primes, but perhaps even circular: Even including pseudo-primes, which are

obviously multitudes, we don't find such a lot of primes within the regions of 10^n with $n > 10^4$, to be able to show by this kind of quasi-empirical evidence, that the usual estimations of the frequency of primes is correct or reliable. But nevertheless, this estimation is in common tests used to get reasonable evaluations of the partially proofs that are necessary to believe in the sufficiency of a primality's probability.

This is especially annoying, as we can get an analytical determination of the proportion of the number of primes to the number of multitudes in N, based on a simple combinatorial formula for combinations with repetition without respect to order. And furthermore, we can exactly determine the next prime within N even without finishing the complete computation just using a respective realm of the gaps in M.

3 The algorithm's formula and an example

n is the number (the cardinality) of primes chosen from N, k is the number of primes chosen for the combinations (products), and k $\leq$ n to avoid doubling. Hence, for the three primes 2,3,5 e.g. $2 \cdot 2$ and $2 \cdot 2 \cdot 2$ are allowed, but 2^4 starts not before 2,3,5,7. The sum that builds M is then the unity-set of all results given by the combinations and multiplication of k with $1 \leq k \leq n$ primes in N.

$$\frac{n}{\sum_{k=1}^{n} \binom{n+k-1}{k}}$$

$$\Leftrightarrow \frac{n}{\sum_{k=1}^{n} \frac{(n+k-1)!}{k!(n+k-1-k)!}}$$

$$\Leftrightarrow \frac{n}{\sum_{k=1}^{n} \frac{(n+k-1)!}{k!(n-1)!}}$$

e.g. for three primes $(2, 3, 5)$ we have a combinatorial set N with 19 members:

$$\frac{3}{\frac{(3+3-1)!}{3!(3-1)!} + \frac{(3+2-1)}{2!(3-1)!} + \frac{(3+1-1)!}{1!(3-1)!}}$$

$$\frac{3}{\frac{(5!}{3!(2)!} + \frac{(4!)}{2!(2)!} + \frac{(3)!}{1!(2)!}}$$

$$\frac{3}{10+6+3}$$

Hence, we get 19 numbers of N with 3 primes, and have a proportion of 3 to 19, or, if we like to look at the disjunct sets, we have 3 primes in P and 16 multitudes in M.

The multitudes for the primes 2,3,5 (without the primes) are
$(2 \cdot 2), (2 \cdot 3), (2 \cdot 5), (3 \cdot 3), (3 \cdot 5),$
$(5 \cdot 5), (2 \cdot 2 \cdot 2), (2 \cdot 2 \cdot 3), (2 \cdot 2 \cdot 5),$
$(2 \cdot 3 \cdot 3), (2 \cdot 3 \cdot 5), (2 \cdot 5 \cdot 5),$

$(3 \cdot 3 \cdot 3), (3 \cdot 3 \cdot 5), (3 \cdot 5 \cdot 5),$
$(5 \cdot 5 \cdot 5 = M$ with $||M|| = 16$.

The uniqueness of the combinatorial results can be understood referring to Euclid and his idea of a proof for the infinity of primes. We use likewise the uniqueness of combinations and the combination of primes to conclude on the uniqueness of results in respect to multiplication. We can show the essence of this argument in a very short proof:

For the trivial claim, that different prime factors have different results, we just use the simple law of multiplication in N without 0, that $a \cdot p \neq b \cdot p$. As there is always at least one different factor in our combinatorial sets if we combine without order and with repetition, we have merely disjunctive results of multiplication in our set M.

To prove this, we just add a common factor to a different base:
$n \cdot a \cdot p \neq n \cdot b \cdot p$ and we can look at the equivalence
$n \cdot a \cdot p = n \cdot b \cdot p \leftrightarrow a = b$
Hence,if we consider a,b,m,n,p:
$n \cdot a \cdot p \neq m \cdot b \cdot p \rightarrow n \cdot a = m \cdot b$ Then $\exists p_1, p_2, p_3, p_4$ such that $n \cdot a = p_1 \cdot p_2 \cdot ... \cdot p_n$ and m$\cdot b = p_3 \cdot p_4 ... \cdot p_m$, such that $p_2 \cdot ... \cdot p_n = p_3 \cdot p_4 ... \cdot p_m$
. If we look at the prime structure, we can divide the equation by the singular primes:
$p_2 \cdot ... \cdot p_n = p_3 \cdot p_4 ... \cdot p_m| : p_m$

As all factors are prime, we can not divide any factor by partialisation. Hence, with every division on the right side one of the factors of the other side has to be deleted, too, e.g.:
$p_2 \cdot ... \cdot p_n = p_3 \cdot p_4 ... \cdot p_m| : p_m$
$p_2 \cdot ... \cdot p_{n-1} = p_3 \cdot p_4 ... \cdot p_{m-1}$
We can repeat this until we arrive at the trivial equation 1=1.
Hence, all prime factors in a set must be equal, if any two of such combinatorial sets are equal. As every combinatorial set has a different factor or at least one or more factors more or less than every other set, all sets are disjunct and the resulting products of the prime-sets all different: 1 to n different prime factors exclude similar results. For this reason, we can take the result of the combinatorial sum as the cardinality of multitudes (products) build by the given prime set (the primes itself are included because of k = 1 allowed).

4 Generalisation

Hence, with $M \cup P = N \setminus \{0, 1\}$, we have a complete surview of the relation of magnitudes of primes to multiples in N for subsets of N that entail succeeding primes. As the combinatorial approach and the law of disjunctiveness of sets do not depend on the special primes that are chosen, we should always get a combinatorial set of multitudes, whose cardinality is calculable based on the cardinality n (in our formula) of used primes p $\in P$. Hence, the relative or absolute frequency of primes to their combinatorial set in N does not depend on the sequence of primes we choose, but merely on their number. The proportion that is given by the special combinatorial formula is valid for every arbitrarily chosen set of primes and equals the result we know from combinatorial mathematics for the special formula given. We have with this equality to combinatorial calculations indeed a simple proof for primality, as the result differs from the usual combinatorial result, when non-primes in P lead to similar products in M and in consequence, less than the necessary number of elements in M. Such a result could be visible soon within a test with the first computations on just two or three chosen elements, that lack cardinality because some factors are twice within the numbers. This allows good and simple random tests of primality, of high primes with some factors that are considered to be probable divisors - but this article tries to promote a more restrictive look at primes and primality.

5 Convergence and the question, whether the number of primes is finite

If we have a look at $n \to \infty$, we see the possibility to raise the magnitude of P with ever higher n (as our magnitude $n = ||P||$) and to conceive nevertheless this magnitude as finite in the sense of convergence to 0 in respect to the number of multitudes. We have, in this way, a kind of 'paradox', then: If we raise n to n+1, we raise the cardinality of primes. But then we *presuppose* that it is always possible to raise n to n+1, even if we show then, that this cardinality ends in respect to the multitudes in N, that are presented by the sums of combinations of primes. How to cope with this? The answer is in a way simple, as the formula is a calculus, an algorithm to build not just multitudes but to find primes, too. Hence, the next prime for the next cardinality n+1 must be selected as a natural number that fills a gap in the set M that itself was generated on a given set P with a cardinality of n primes. Before a paradox arises, we will have arrived at a point, where no more gaps for further primes are in the line of naturals in M, as N (always add 1 and 0, if you like), is then generated as unity-set of primes and their multitudes. In short, it will depend on the question whether the gaps between the multiples vanish by the raising number of multiples that emerge with every new combination. If the gaps vanish, there won't be any more primes to raise n up to n+1. With the laws of analysis, we should imagine this situation to arrive when the sequence's value has converged to 0.

Though I confess that further insight into the structure of the combinatorial sets might be of use, the convergence should nevertheless be conceived to show, that with the raising number of prime factors and multitudes, the given finite primes can close gaps to become neighbours, like $2 \cdot 2 \cdot 5 = 20$, $3 \cdot 7 = 21$, $2 \cdot 11 = 22 \ldots \in N$. A list for the primes 2,3,5,7,11 is given in the appendix to this article.

The special kind of combinatorial structure we propose seem to guarantee that the prime factors distribute products with different numbers of factors, and thus cover succeedingly the whole region.

Especially, if anybody likes to conjecture with Euclid, that the faculty of the multitudes given will build an interval that entails a further prime that is not already in P, we can argue against this, that the faculty $m!$ is already in M and its gaps filled with combinations between m and m^m, as the faculty is just the product of every different element of the base set P. Euclid's argument is weak in the point that it just offers no special insight into the interval and how this can be widened and filled with multitudes by the finite set of primes. Hence, what he shows is that the faculty operation on one finite set of primes is insufficient to find all primes, but not, that the repeating recurrence on widened finite sets and the combinatorial approach do not lead to a set M without prime-gaps. It seems as if the attempt to detect a further prime in the interval between the faculty's highest prime m and the result of the faculty had dissembled the potency of multitudes within this very strange region of nearly infinite, but still finite, numbers of primes n.

Hence, Euclid's proof shows, that the faculty of primes for the finite set of primes we choose is insufficient to build the natural numbers (even without 0 and 1). What our formula shows, is that the algorithm based on finite sets of primes can actually be sufficient, when we - of course on the way to infinity - need not any more primes or any primes beyond our base set P. In a way, the formula is exactly acting on Euclidian finite sets of primes such that it allows to reach complete sequences of natural numbers with a cardinality n to ∞. The answer against Euclid is to use not one finite set, but a nearly endless number of finite sets, and to use all the combinations of the finite selection of primes to construct N. The criteria of finitness should be, that the algorithm stops when no more prime gap is given. To take this to be the case, is at the state of the art justified by an argument from analysis, that shows the convergence of the algorithm's series to 0.

The convergence, again, explains and proves, that we have a set M of multitudes, that entails nearly infinite members, hence m^m with m $\in M$ is possible and higher than $M!$, and, as the point of convergence is reached, all gaps within M are already closed by the n primes in P and their multitudes. N will go on then as a neighbour-sequence of multitudes, with powers allowed that succeed the cardinality of the primes. That within the current Mersenne primes in regions of $10^{1,000,000}$, we have a good hint to believe that such a situation is in the advent, when primes are reduced to powers of 2^n +/- 1 and all other gaps can be filled with multitudes. As the double $2 \cdot (p_{n+1})$ of the first gap in M, that is always prime, is the upper limit of the gaps in M that are necessarily prime, the approximation of highest primes to such a power of 2 could show that the prime after the first gap, p_{n+2}, shifts more and more towards the upper limit $2(p_{n+1})$, what might mean, that at a certain stage of $n \to \infty$, the occurence of such a second gap might end.

6 Proof for the convergence of the series

$\frac{n}{\sum_{k=1}^{n} \binom{n+k-1}{k}}$

as the algorithm for the set M of primes and their combinatorial products.

We know $\frac{1}{\sum_{k=1}^{n} \binom{n+k-1}{k}} \leq \frac{1}{\sum_{k=1}^{n} \binom{n}{k}}$.

and we show $\frac{1}{\sum_{k=1}^{n} \binom{n}{k}} \leq \frac{1}{2^n}$:

$\frac{1}{\sum_{k=1}^{n} \binom{n}{k}} = \frac{1}{\binom{n}{1}+\binom{n}{2}+...+\binom{n}{n}} = \frac{1}{2^n-1}$

As $\frac{n}{\sum_{k=0}^{n} \binom{n+k-1}{k}} \leq \frac{n}{\sum_{k=1}^{n} \binom{n+k-1}{k}} \leq \frac{n}{\sum_{k=0}^{n} \binom{n}{n-k}}$ we can show

$\frac{n}{\sum_{k=0}^{n} \binom{n}{n-k}} \leq \frac{n}{2^n}$ and that $\frac{n}{2^n}$ converges:

With the binominal rule for $n \geq 2$ we have:

$2^n = (1+1)^n = 1 + n + \frac{n(n-1)}{2} + ... \geq \frac{n(n+1)}{2} \geq \frac{n^2}{2}$.

Hence, $\frac{n}{2^n} \leq \frac{2n}{n^2} \leq \frac{2}{n}$, with $|\frac{2}{n}| < \epsilon$ for nearly all n.

So, we have a formula that gives us the distribution of primes within their multitudes, and with both disjunct sets of P and M the complete set N. The formula is not only useful to determine the specific $\pi(x)$ on the base of an absolute frequency, but allows us even to determine exactly the number of primes for any finite number of natural numbers, as the single members of the sequence are uniquely determined: with a given number of primes, one and only one number of multitudes in the combinatorial structure can emerge. We just have to shift the attempt to calculate numbers of primes within an open interval at the first place. If we want to get an exact information about such an interval, we just have to look at the respective sets of multitudes first.

As the relation of multitudes is given by two laws, the rule for combinations with repetition $\binom{n+k-1}{k}$, and the rule of N to be the unity-set of the set of primes P and the set of multitudes of primes M, the calculated relation of numbers of primes and number of multitudes is far more reliable than any estimation that is relative to an arbitrarily chosen interval or magnitude of natural numbers.

The relative frequency of primes within a realm of N is for every set P:

$\frac{||P||}{\sum_{k=1}^{n}\binom{n+k-1}{k}} \leq \frac{||P||}{\sum_{k=1}^{n}\binom{n}{k}}$,

e.g. for $||P|| = 7$:

1716 + 924 + 462 + 210 + 84 + 28 + 7 = 3431

or $||P|| = 70$:

70 + 2485 + 59640 + 1088430 + 16108764 + 65 further values,

up to combinations of all 70 elements of P with repetition, that means, ${p_{70}}^{70}$ included.

We should remember, that N is definitely a unity of primes and their multitudes. Hence, the relation that becomes obvious here is the calculable relation that is given in N. The great advantage in comparison to probabilistic estimations on arbitrary subsets with an arbitrary cardinality is, that the cardinality of the combinatorial set M *follows* by the capability of n primes to build products with restricted powers. Hence, if we like to become such metaphysical that we talk about the existence of numbers in N, the combinatorial calculation shows the cardinality's proportion of existing primes to existing multitudes. We have already explained that N is structured in this way. Hence, we have no good reason to look at the frequency of primes and the distribution of primes within N otherwise.

7 Some suggestions about the prime-gaps in M

Fortunately,as multitudes explode on prime combinations, the gaps within M are quite narrow and easy to determine. The first gap after ordering the elements of M by size is always a prime gap, and some further primes are between this new prime and its double, as this $2 \cdot p_{n+1}$ will be the first gap filled with the next combinatorial calculation.

In this way, we can give a more precise approach to the problem than Euclid, though we do not necessarily believe that such gaps will always exist. However, if we keep the already given multitudes for further constructions, we have just to add the combinatorial products that emerge by adding the new prime to the sets, and widening these with one more factor. A small basis of primes and a small selection of combinations is sufficient to get narrow intervals.
Within these intervals the gaps can be checked by progressing the procedure or, if we think this is sufficient for a case, by random checks on the base of small selections of primes in P, and the prime to be checked in P. The combinations

on these random bases should fit the regular cardinality, as this guarantees coprimality. But as my attempt is an algorithm without principles of random, and not an estimation of primes in N, but a calculation, I think it can be refined to describe and calculate the respective sequences of prime-gaps generally, though this is not subject of this article.

8 Advantages over common methods

However the weaknesses of Euclid's proof are repaired within the contemporary literature, all proofs depend on the uniqueness of division of any natural number, where the discussion focuses on. It is probably wrong to look at this as a problem, as we can easily show, that for the interesting subsets of N, we can show how they are generated as disjunctive, therefore uniquely determined products of some primes. After having refered to the problems in Euclid's arguments for an infinity of primes (like the unprovability of infinite primes for every modular subset in N), and to alternative, contemporary proofs for the claim of infinity, Steuding 2001 explains, that the still not completely proved uniqueness of composition experiences an analytical approach with the ζ-function (Steuding 2001: 16). An investigation into the argument leads to the weak law of large numbers $P(p|n)$:

$\lim_{x \to \infty} \frac{[x/p]}{[x]} = \frac{1}{p}$

It should follow by this law, that the probability of n arbitrarily chosen natural numbers are not divisible with a prime p (are coprime to this prime p) equals $(1 - (\frac{1}{p^n})$.

This leads to the probability, that n arbitrarily chosen natural numbers are coprime (do not have common factors):

$P(ggT(m_1, ..., m_n) = 1) = \Pr(1 - (\frac{1}{p^n})) = \frac{1}{\zeta(n)}$

$= \frac{6}{\pi^2} = 0.608(35)$

Steuding (2001:39, log is ln) shows that the highest value of the $\pi(x)$ factor (estimated as 2.6) is 3:
$|\sum_{p \le x} \frac{\log p}{p} - \log x| < 3$.(Steuding 2000: 20), where $\pi(x)$ is in the ζ-function:

The zeta-function leads to a more precise number of primes (Steuding 2001: 39) and supports different elementary and analytical proofs of the Gaussian assumption. It gains its value as a kind of combination of Euler's number theoretical argument, that the roots of the ζ-function equal the distribution of primes within the cosets (Steuding 2001: 36), and the Gaussian assumption. The constant c within the ζ-function then shows the regular distribution of primes in the cosets and allows finally to prove the 'Primzahlsatz' with the result of

$\pi(x) \approx \frac{n \log n}{\log n \log n}$ (Steuding 2001: 45)

The argument seems to rely on an analogy to the asymptotical values of the ζ-function and supports not exactly a generalisation of the Gaussian assumption, but the underlying assumption of a regular distribution of primes within the cosets. Hence, it looks vulnerable to a rebuilding of primes and N based on primes and their multitudes.

However, with a strong simplification, the relative frequency of primes in N is

$\sum_{n=2}^{\infty} = \frac{1}{2.6n} \cdot 10^n$

We show the quite obvious fact that this series diverges:

With $\frac{1}{2.6n} \cdot 10^n$:

$10^n \frac{1}{2.6n} = (1+9)^n \frac{1}{2.6n}$

$= \frac{9^9(n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)(n-8)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 2.6n}$ for $n \geq 9$ we have

$(n-9) \geq \frac{8}{9}n,$

$(n-8) \geq \frac{7}{9}n,$

$(n-7) \geq \frac{6}{9}n,$

$(n-6) \geq \frac{5}{9}n,$

$(n-5) \geq \frac{4}{9}n,$

$(n-4) \geq \frac{3}{9}n,$

$(n-3) \geq \frac{2}{9}n,$

$(n-2) \geq \frac{1}{9}n,$

hence, $\frac{9^9(n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)(n-8)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 2.6n} \geq \frac{n^8}{2.6}$

and $\geq n$. Hence, with $n \to \infty$ the series is divergent (Duma 1984).

Additionally, we can look at the absolute magnitudes of primes which are added with every succeeding n in 10^n:

A chart gives the following data:

25 primes in $10^2| \cdot 6.72$

$= 168$ in $10^3| \cdot 7.31$

$= 1229$ in $10^4| \cdot 7.80$

9592 in $10^5| \cdot 8.18$

78498 in $10^6| \cdot 8.466$

664579

5761455

50847534

455052511

4118054813

The primes seem to tend to grow with a multitude of 9 for every added power of 10^n. Within every new realm of 10^n we might think of having

$\frac{1}{n \cdot 2.6} \cdot 10^n - \frac{1}{(n-1) \cdot 2.6} \cdot 10^{(n-1)}$ as number of new primes, what is about $\frac{(9 \cdot n - 10) \cdot 10^{n-1}}{2.6 \cdot n \cdot (n-1)}$,

To check how this series progresses, we can look at the factorial form:

$(\frac{9}{2.6 \cdot (n-1)}$ - $\frac{10}{2.6 \cdot n \cdot (n-1)} \cdot 10^{n-1}$,

and as $\frac{10}{2.6 \cdot n \cdot (n-1)}$ is faster convergent than $\frac{9}{2.6 \cdot (n-1)}$,

because of the higher power of n, we see that the difference to the schema for the complete frequency declines, hence, the amount of new primes rises, and such, the proportion keeps the line. At least it should, as we accept this schema. But with the evidence given by the combinatorial argument and respecting the weak probability based confirmation of primality, this estimation seems far too high.

If we have a look at Mersenne primes which are proved by a kind of Euclidian algorithm and probably completely, these are far too less than we would need to confirm the Gaussian assumption for high n. The Gaussian assumption has its name because it was an observation based on quite small numbers - and faces now a lack of evidence given by really high primes, e.g. the upper 1000. The discovered high Mersenne primes change in the first or second digit of the exponent, but they should succeed in every digit of the exponent. Hence, the distances don't fit to the Gaussian suggestion at all.

Third, the identification of Non-Mersenne primes in usual tests is not just unreasonably based on magnitudes of the realm, but circular in respect to the estimation of the primes distribution in N. Hence, if at all taken into respect, the realm is considered to fit into the $\frac{1}{2.6 \cdot n}$ schema, that can not be proved by the evidence given by numbers in N itself.

Looking at the usual test of primality, we have to notice a circularity in the methods not just for proving primality, but for the evidence we get for the reasonableness of the $\frac{1}{2.6 \cdot n}$ assumption, too. The Miller-Rabin-Test, which is most often used for primes of intermediate size with powers like 10^{1000}, is based on the probability assumption:

for a natural number n and a set A:=$\{a|$ n is a strong pseudo-prime relative to base a$\}$ with an approximation of

$A \leq \frac{1}{4} \cdot \phi(n) \Leftrightarrow A \cdot \phi(n) \leq \frac{1}{4}$.

(primzahlen.zeta24.com)

The probability of a failure (that the result does not fit the hypothesis, not, that the hypothesis is generally misled) is reduced by repetitions, about k = 100 for a usual test. By this method, the probability of a failure is reduced to $\frac{1}{4}_k$ with the result of a probability of 10^{61} = 0,00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 1. This seems impressive, but it is not. The result is just, that the number is a prime or a pseudoprime, that is, a number which is divisible into two large primes. Hence, first at all, what we judge to be primes are not primes in the important sense of the restrictive distinction between primes and multitudes. A pseudoprime is definitely a multitude of a prime and would destroy a combinatorial system completely.
Furthermore, a 10^{61} probability of a failure is high relative to the magnitude of the realm we are talking about - if we prove primes with about 3000 digits, what is the intermediate size of well known primes, a calculation on 100 bases and a probability of 10^{61} seems far too less.
The circularity of this test is obviously, that it presupposes to have already a well and formally proved, reliable knowledge about the distribution and frequency of primes within the realm of N. Hence, with proof-methods like the Miller-Rabin test, we seem to confirm these assumptions, but really depend on them as we don't leave the realm of probabilities. To judge about the proportionality of primes to 'other' natural numbers (their multitudes) by using enumerations of primes within arbitrarily chosen intervals of N seems hence completely muddling, and should give way to the proposed computational algorithm that is

founded in the abstract relation of primes and their multitudes within N.

9 An algorithm for the detection of primes and the calculation of their relative frequency in subsets of N

I think the amount of recapitulation and precision of a mixture of intuition, observation and the authority of Gauss in the common literature is not really necessary to get an approximation to the proportion of primes and naturals and the distribution of primes in N. To discuss the problem in the common way creates itself obstacles by its linear view of numbers - that we should ask for the cardinality and distribution of primes by looking at a linear constructed set of naturals, built by their own recursive rule $n := n+1$. This obviously dissembles the way how primes and their multitudes arrange with each other in N.

With looking at primes first, we can build multitudes for M, an ordered structure of neighbours in $M \cup P$ and a clear structure of the distribution by the combinatorial operations given with

$$\frac{n}{\sum_{k=1}^{n} \frac{(n+k-1)!}{k!(n-1)!}}$$

with n = $||P||$, $k \leq n$ and the computation of multiplication on the chosen combinations of numbers:

For $P = 2, 3$:
$(2, 2) = 2 \cdot 2 = 4$
$(2, 3) = 2 \cdot 3 = 6$
$(3, 3) = 3 \cdot 3 = 9$
$(2) = 2$
$(3) = 3$
.

Hence we get a set $2, 3 \cup 4, 6, 9$
and the gaps 5,7,8, which are just interesting because they should entail the next prime, (if there is still one). We can assume that the lowest number with the set of gaps is the candidate for primality, and that the specific magnitudes of primes in the gap-sets should be determinable.

We start with the next prime-set:
For $P = 2, 3, 5$:

Using our formula, we get 16 numbers of N with 3 primes, and have a proportion of 3 to 19, or, if we like to look at the disjunct sets, we have 3 primes in P and 16 multitudes in M.

(The multitudes for the primes 2,3,5 without the primes are
$(2 \cdot 2), (2 \cdot 3), (2 \cdot 5), (3 \cdot 3), (3 \cdot 5), (5 \cdot 5)$,
$(2 \cdot 2 \cdot 2), (2 \cdot 2 \cdot 3), (2 \cdot 2 \cdot 5)$,
$(2 \cdot 3 \cdot 3), (2 \cdot 3 \cdot 5), (2 \cdot 5 \cdot 5)$,
$(3 \cdot 3 \cdot 3), (3 \cdot 3 \cdot 5), (3 \cdot 5 \cdot 5)$,
$(5 \cdot 5 \cdot 5) = M$ with $||M|| = 16$.

Including the primes 2,3,5 we have 19 numbers, the result of the combinatorial sum:

2,3,4,5,6,8,9,10,12,15,18,20,25,27,30,45,50,75,125

again with gaps that are filled in the next step, when we include 7 as prime and a multiplication of up to four numbers. Again the lowest gap is the next prime.

We can proceed with this algorithm endlessly, and will get a determined and simple criteria whether a number is a prime or not: If we get a number from the gap-list twice in the set of multitudes, we have multiplied a multitude. Hence, the provability of magnitudes, that is provided by the formula, is sufficient for decidability, whether a set is correctly calculated by adding one and only one prime to P. E.g., if we use 4 instead of 5 for $P = 2, 3, 4$, we get

$(2 \cdot 2), (2 \cdot 3), (2 \cdot 4), (3 \cdot 3), (3 \cdot 4), (4 \cdot 4)$,
$(2 \cdot 2 \cdot 2), (2 \cdot 2 \cdot 3), (2 \cdot 2 \cdot 4), (2 \cdot 3 \cdot 3)$,
$(2 \cdot 3 \cdot 4), (2 \cdot 4 \cdot 4), (3 \cdot 3 \cdot 3), (3 \cdot 3 \cdot 4)$,
$(3 \cdot 5 \cdot 4), (4 \cdot 4 \cdot 4)$

$*4*, 6, *8*, 9, 12, 16, *8*$,
$12, 16, 18, 24, 32, 27, 48, 60, 64$. As we have multiplied multitudes, we have a double number 8 in the set and a repetition of one of our pseudo-primes, and hence not the correct cardinality of 16 in the combinatorial set M.

10 The gaps and suggestions for short procedures

A possible conjecture to our combinatorial algorithm might be, that we need a special reason not to compare the given number of primes with the infinity of powers they may have. This is justified by the restriction to defined methods of computation, especially multiplication within a power that is defined by the cardinality of primes we operate on. We can nevertheless multiply given multitudes endlessly, even if primes might end, and though the concepts of growth and identity of numbers are perhaps different within this realm, could probably not fit into our concept of existence or fit even better in current concepts of physics.

11 References

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12 Appendix: List of multitudes of primes (2,3,5,7,11)

2	54	231	660	1815	6125
3	55	242	675	1875	6655
4	56	243	686	1925	6875
5	60	245	693	2058	7203
6	63	250	700	2079	7546
7	66	252	726	2156	7623
8	70	264	735	2178	7986
9	72	270	750	2205	8085
10	75	275	770	2310	8470
11	77	280	825	2401	8575
12	80	294	847	2420	9075
14	81	297	875	2450	9317
15	84	300	882	2475	9625
16	88	308	891	2541	11319
18	90	315	924	2625	11858
20	98	330	945	2662	11979
21	99	343	968	2695	12005
22	100	350	980	2750	12705
24	105	363	990	3025	13310
25	108	375	1029	3087	13475
27	110	378	1050	3125	14641
28	112	385	1078	3234	15125
30	120	392	1089	3267	16807
32	121	396	1100	3388	17787
33	125	405	1125	3430	18634
35	126	420	1155	3465	18865
36	132	440	1210	3630	19965
40	135	441	1225	3675	21175
42	140	450	1250	3773	26411
44	147	462	1323	3850	27951
45	150	484	1331	3993	29282
48	154	490	1372	4125	29645
49	162	495	1375	4235	33275
50	165	500	1386	4375	41503
54	168	525	1452	4802	43923
	175	539	1470	4851	46585
	176	550	1485	5082	65219
	180	567	1540	5145	73205
	189	588	1575	5324	102487
	196	594	1617	5390	161051
	198	605	1650	5445	
	200	616	1694	5775	
	210	625	1715	5929	
	220	630	1750	6050	
	225				